MATH 245 F20, Exam 3 Questions

(60 minutes, open book, open notes)

1. Freebie.
2. Let $S = \{x \in \mathbb{Z} : \exists y \in \mathbb{Z},\ x = 6y + 5\}$ and $T = \{x \in \mathbb{Z} : \exists y \in \mathbb{Z},\ x = 2y + 1\}$. Prove or disprove that $S = T$.
3. Let R, S, T be sets. Prove that $(R \setminus S) \setminus T \subseteq R \setminus (S \setminus T)$.
4. Let $S = \{x\}$. Find a set T that simultaneously satisfies all of the following properties: $S \not\subseteq T$, $2^S \in T$, $2^S \subseteq T$, $S \times 2^S \subseteq T$. Be very careful about notation.
5. Prove or disprove: For all sets S, U with $S \subseteq U$, we have $2^S \cup 2^{(S^c)} = 2^U$.
6. Let A, B, C be sets. Prove that $A \times (B \setminus C) = (A \times B) \setminus (A \times C)$.
   Note: Do not just cite a theorem.
7. Let S be the set of letters in your name (choose first or last). Find a relation R on S that is not reflexive, not irreflexive, not symmetric, not antisymmetric, not trichotomous, and not transitive. Give your relation as a directed graph, and fully justify each of these properties.
8. Let S be a set, $T \subseteq S$, and R a reflexive relation on S. Prove that $(R|_T)^+$ is reflexive.